Fall 2022 MAT 206.5 Final Review

ONLY SCIENTIFIC CALCULATOR PERMITTED FOR THE FINAL EXAM

1. (i) Given $\theta = \frac{7\pi}{6}$,
   a. Convert θ to degrees. Review
   b. Draw θ in the coordinate plane. Review
   c. Name two angles, one positive and one negative, that are coterminal to θ. Review
   d. Determine the reference angle $\hat{\theta}$. Review
   e. Find the exact value of $\cos\theta$. Review
   f. Find the exact value of $\cot\theta$. Review

   (ii) Given $\theta = -\frac{\pi}{3}$
   a. Convert θ to degrees. Review
   b. Draw θ in the coordinate plane. Review
   c. Name two angles, one positive and one negative, that are coterminal to θ. Review
   d. Determine the reference angle $\hat{\theta}$. Review
   e. Find the exact value of $\sin\theta$. Review
   f. Find the exact value of $\sec\theta$. Review

2. Review the below topic here.

   (i) If $\cos\theta = -\frac{\sqrt{2}}{3}$, sketch θ in the coordinate plane, and find $\tan\theta$ and $\csc\theta$, where θ terminates in the second quadrant. Give the exact values.

   (ii) If $\cot\theta = \frac{\sqrt{5}}{4}$, sketch θ in the coordinate plane, and find $\sin\theta$ and $\sec\theta$, where θ terminates in the third quadrant. Give the exact values.

   (iii) If $\sin\theta = -\frac{\sqrt{6}}{4}$, sketch θ in the coordinate plane, and find $\sec\theta$ and $\cot\theta$, where θ terminates in the fourth quadrant. Give the exact values.

   (iv) If $\sec\theta = \frac{7}{3}$, sketch θ in the coordinate plane, and find $\csc\theta$ and $\tan\theta$, where θ terminates in the fourth quadrant. Give the exact values.

   (v) If $\tan\theta = -\frac{\sqrt{10}}{2}$, sketch θ in the coordinate plane, and find $\csc\theta$ and $\sec\theta$, where θ terminates in the second quadrant. Give the exact values.

3. Sketch a graph of the given angle and evaluate the value of a trigonometric function of the angle without using a calculator. Give the exact values. Review this topic.

a) $\tan(210°)$

b) $\cos\left(\frac{2\pi}{3}\right)$

c) $\sin\left(-\frac{9\pi}{4}\right)$

d) $\cot(-150°)$

e) $\sec\left(\frac{17\pi}{6}\right)$

f) $\csc(-630°)$

4. (i) Given $f(x) = x^2 - 4$, $g(x) = \sqrt{3 - x}$, Find the following: Review this topic.

a) The domain of $f(x)$. Write the answer in interval notation.
b) The domain of $g(x)$. Write the answer in interval notation.
c) Evaluate $(f \circ g)(x)$ and simplify.
d) The domain of $(f \circ g)(x)$. Write the answer in interval notation.

(ii) Given $g(x) = \frac{1}{x-4}$, $h(x) = x^2 - 16$, Find the following:

a) The domain of $g(x)$. Write the answer in interval notation.
b) The domain of $h(x)$. Write the answer in interval notation.
c) Evaluate $(h \circ g)(x)$ and simplify.
d) The domain of $(h \circ g)(x)$. Write the answer in interval notation.

(iii) Given $f(x) = x^2 - 3$, $g(x) = \sqrt{x + 2}$, Find the following:

a) The domain of $f(x)$. Write the answer in interval notation.
b) The domain of $g(x)$. Write the answer in interval notation.
c) Evaluate $(f \circ g)(x)$ and simplify.
d) The domain of $(f \circ g)(x)$. Write the answer in interval notation.

(iv) Given $f(x) = \frac{1}{x-7}$, $h(x) = \sqrt{3-x}$, Find the following:

a) The domain of $f(x)$. Write the answer in interval notation.

b) The domain of $g(x)$. Write the answer in interval notation.

c) Evaluate $(f \circ h)(x)$ and simplify.

d) The domain of $(f \circ h)(x)$. Write the answer in interval notation.

(v) Given $f(x) = x - 5$, $g(x) = \log_2(x+3)$, Find the following:

a) The domain of $f(x)$. Write the answer in interval notation.

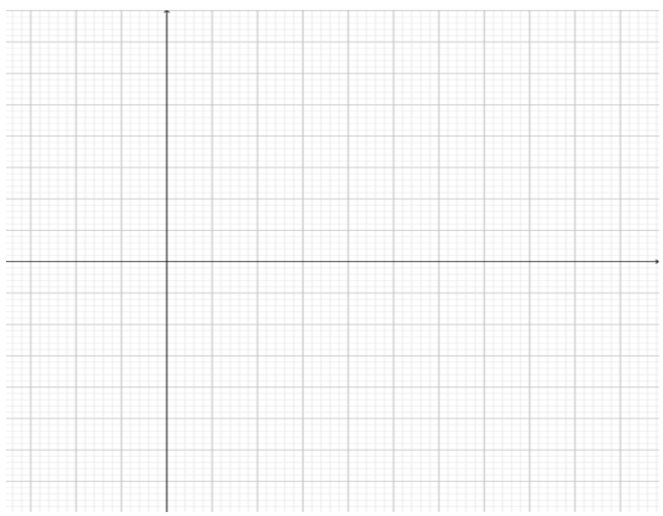
b) The domain of $g(x)$. Write the answer in interval notation.

c) $(g \circ f)(x)$ and simplify.

d) The domain of $(g \circ f)(x)$. Write the answer in interval notation.

5. Review graphing sine/cosine functions here.

(i) Given $f(x) = 2\cos\left(\frac{x}{4}\right)$;



a. What is the period of ***f(x)***?

b. What is the amplitude of ***f(x)***?

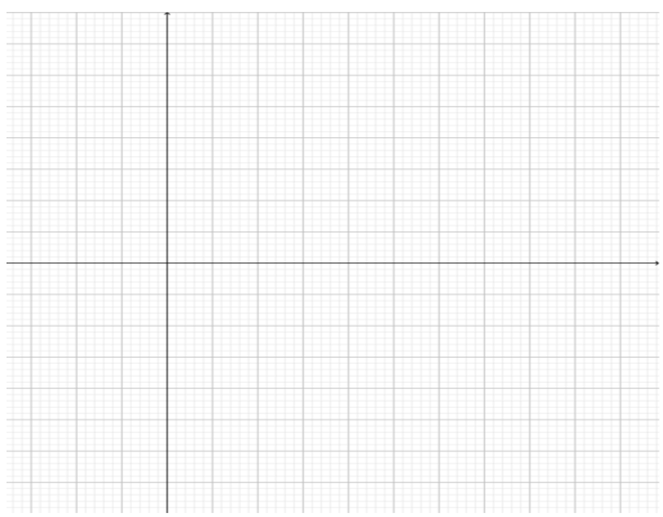
c. What is the phase shift of ***f(x)***?

d. What is the equation of midline of ***f(x)***?

e. What are 5 key points for one period of ***f(x)***?

f. Sketch a graph of ***f(x)*** for two full periods.

(ii) Given $g(x) = -\frac{1}{2}\cos(5x) + 1$,



a. What is the period of ***f(x)***?

b. What is the amplitude of ***f(x)***?

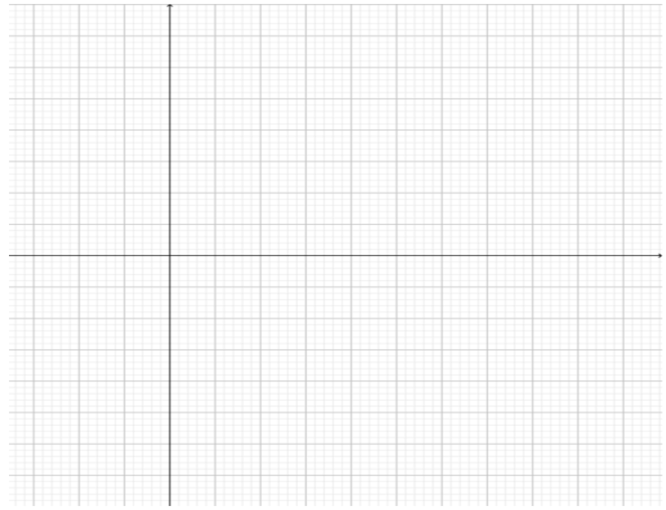
c. What is the phase shift of ***f(x)***?

d. What is the equation of midline of ***f(x)***?

e. What are 5 key points for one period of ***f(x)***?

f. Sketch a graph of ***f(x)*** for two full periods.

(iii) Given $f(x) = 4\cos\left(3x - \frac{\pi}{2}\right) - 1$;



a. What is the period of ***f(x)***?

b. What is the amplitude of ***f(x)***?

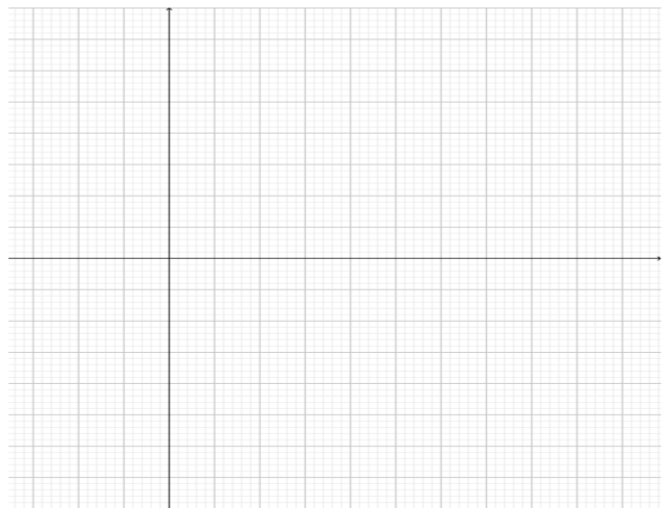
c. What is the phase shift of ***f(x)***?

d. What is the equation of midline of ***f(x)***?

e. What are 5 key points for one period of ***f(x)***?

f. Sketch a graph of ***f(x)*** for two full periods.

(iv) Given $f(x) = -3\sin\left(\frac{x}{2}\right)$;



a. What is the period of ***f(x)***?

b. What is the amplitude of ***f(x)***?

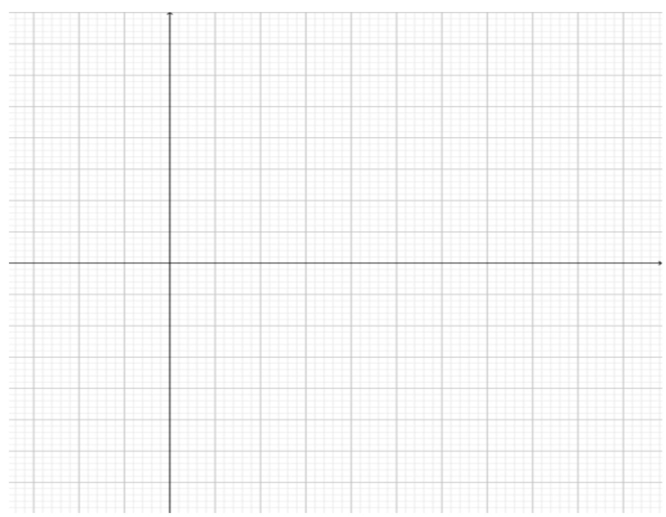
c. What is the phase shift of ***f(x)***?

d. What is the equation of midline of ***f(x)***?

e. What are 5 key points for one period of ***f(x)***?

f. Sketch a graph of ***f(x)*** for two full periods.

(v) Given $f(x) = \frac{1}{2}\sin(5x) + 3$;



a. What is the period of ***f(x)***?

b. What is the amplitude of ***f(x)***?

c. What is the phase shift of ***f(x)***?

d. What is the equation of midline of ***f(x)***?

e. What are 5 key points for one period of ***f(x)***?

f. Sketch a graph of ***f(x)*** for two full periods.

(vi) Given $f(x) = 5\sin\left(4x - \frac{\pi}{2}\right) + 1$;

a. What is the period of ***f(x)***?

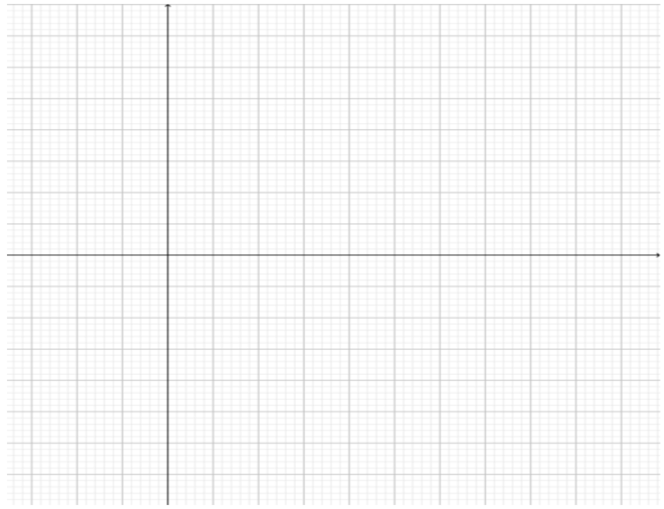
b. What is the amplitude of ***f(x)***?

c. What is the phase shift of ***f(x)***?

d. What is the equation of midline of ***f(x)***?

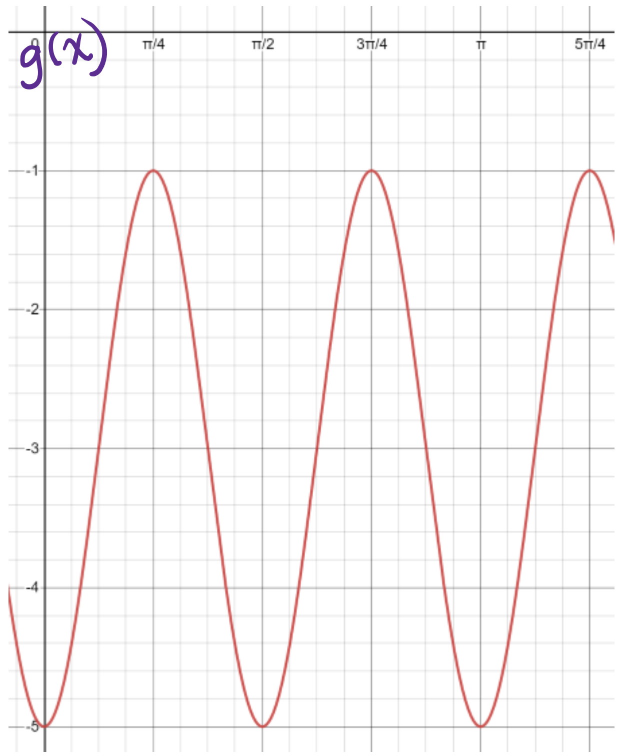
e. What are 5 key points for one period of ***f(x)***?

f. Sketch a graph of ***f(x)*** for two full periods.



6. Review graphing sine/cosine functions here.

(i) The graph below ***g(x)*** is a transformation of the graph of $f(x) = \sin(x)$. There are no reflections in this graph from ***f(x)***.



a. What is the period of ***g(x)***?

b. What is the amplitude of ***g(x)***?

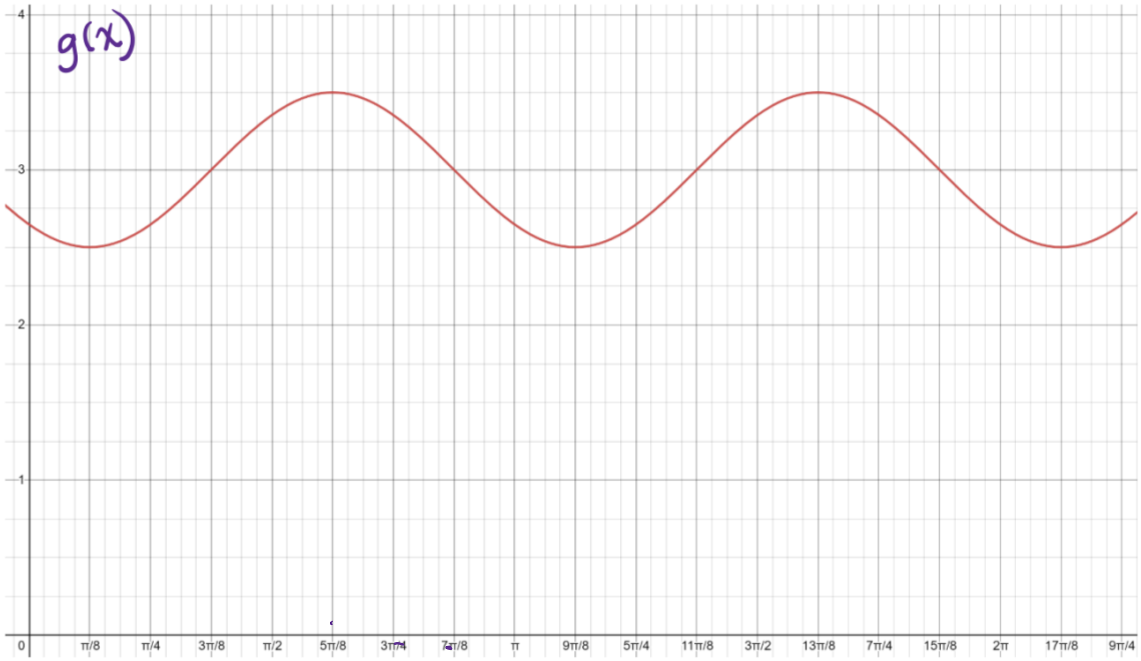
c. What is the phase shift of ***g(x)***?

d. What is the equation of midline of ***g(x)***?

e. What are 5 key points for one period of ***g(x)***?

f. What is the equation of ***g(x)***?

(ii) The graph below ***g(x)*** is a transformation of the graph of $\boldsymbol{f(x) = \cos(x)}$. There are no reflections in this graph from ***f(x)***.



a. What is the period of ***g(x)***?

b. What is the amplitude of ***g(x)***?

c. What is the phase shift of ***g(x)***?

d. What is the equation of midline of ***g(x)***?

e. What are 5 key points for one period of ***g(x)***?

f. What is the equation of ***g(x)***?

7. (i) Given $f(x) = 2x^2 - 3x$, find $\frac{f(-3+h)-f(-3)}{h}$.

(ii) Given $g(x) = 2x + 7$, find $\frac{g(-4+h)-g(-4)}{h}$.

(iii) Given $f(x) = \frac{1}{x+6}$, find $\frac{f(x+h)-f(x)}{h}$

(iv) Given $f(x) = 4$, find $\frac{f(-5+h)-f(-5)}{h}$

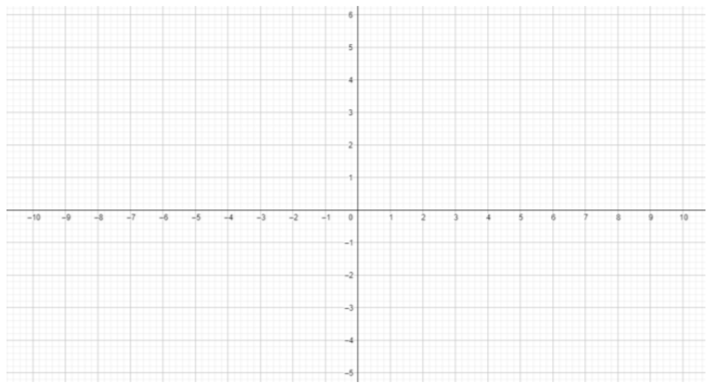
(v) Given $g(x) = 4x - x^2$, find $\frac{g(x+h)-g(x)}{h}$

(vi) Given $f(x) = \frac{5}{x-3}$, find $\frac{f(7+h)-f(7)}{h}$

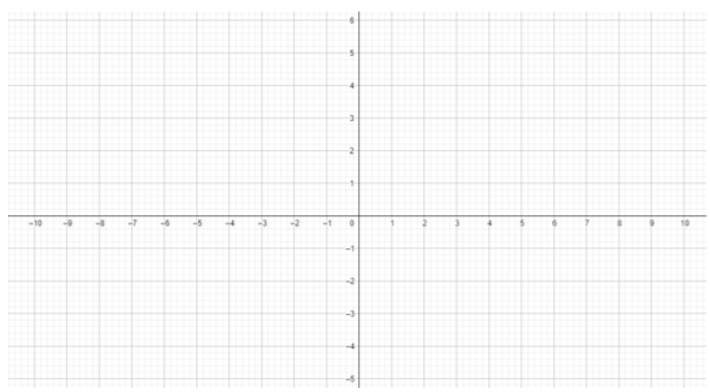
Review difference quotients here.

8. Review graphing rational functions here.

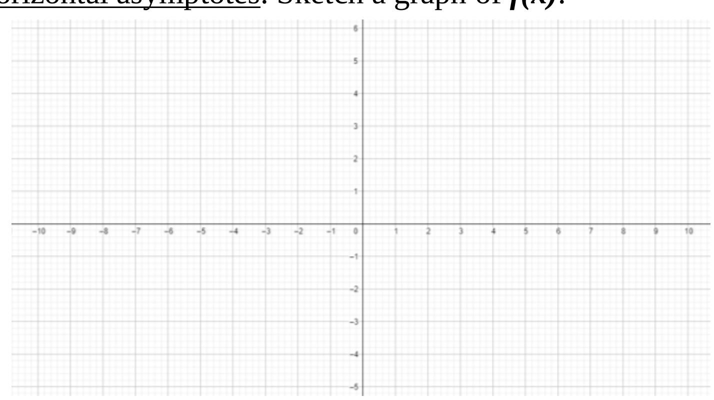
(i) Find the domain of $f(x) = \frac{x^2 - 25}{x^2 + 3x - 40}$, and identify any discontinuous point(s) (holes), vertical and/or horizontal asymptotes. Sketch a graph of ***f(x)***.



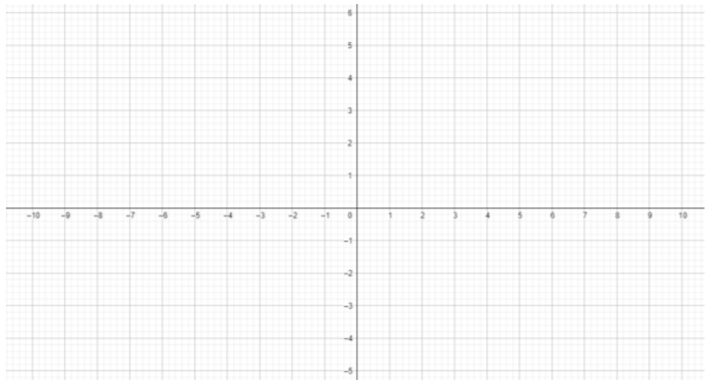
(ii) Find the domain of $f(x) = \frac{x + 4}{x^2 - x - 2}$, and identify any discontinuous point(s) (holes), vertical and/or horizontal asymptotes. Sketch a graph of ***f(x)***.



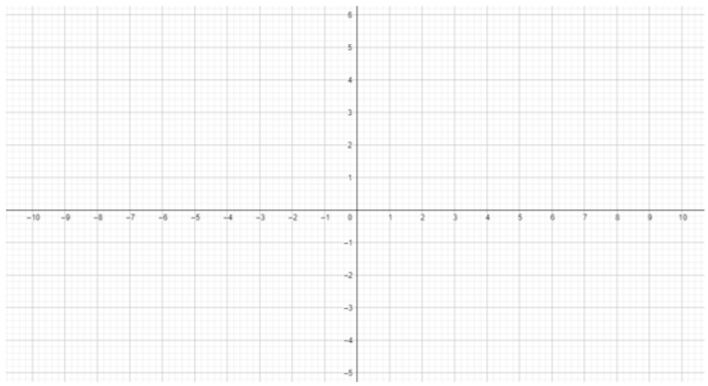
(iii) Find the domain of $f(x) = \frac{2x^2 - 6x}{x^2 + 3x - 18}$, and identify any discontinuous point(s) (holes), vertical and/or horizontal asymptotes. Sketch a graph of ***f(x)***.



(iv) Find the domain of $f(x) = \frac{2x+4}{x^2+8x+12}$, and identify any discontinuous point(s) (holes), vertical and/or horizontal asymptotes. Sketch a graph of ***f(x)***.



(v) Find the domain of $f(x) = \frac{-3x^2+27}{x^2-7x+12}$, and identify any discontinuous point(s) (holes), vertical and/or horizontal asymptotes. Sketch a graph of ***f(x)***.



9. Write the partial fraction decomposition of the rational expression. Check your result algebraically.

a) $\frac{1}{x^2+x}$ b) $\frac{3}{x^2-3x}$ c) $\frac{5}{x^2+x-6}$ d) $\frac{x+1}{x^2-x-6}$ e) $\frac{2x-3}{(x-1)^2}$

Review partial fractions here.

10. Review synthetic division here.

(i) Use synthetic division to find all real solutions of $x^3 - 19x - 30 = 0$.

(ii) Use synthetic division to find all real solutions of $y^3 + y^2 - 17y + 15 = 0$.

(iii) Use synthetic division to find all real solutions of $a^3 - 21a - 20 = 0$.

(iv) Use synthetic division to find all real solutions of $2x^3 - 13x^2 + 5x + 6 = 0$.

11. Review finding zeros of a polynomial function here.

(i) Find all zeros of $f(x) = 3x^3 - 2x^2 + 48x - 32$ given that $\frac{2}{3}$ is a zero of $f(x)$.

(ii) Find all zeros of $h(x) = 3x^4 + 7x^3 - 25x^2 - 63x - 18$ given that -2 is a zero of $h(x)$.

(iii) Find all zeros of $h(x) = 4x^4 + 11x^3 - 67x^2 - 176x + 48$ given that -3 is a zero of $h(x)$.

(iv) Find all zeros of $g(x) = 2x^4 - 11x^3 - 3x^2 + 44x - 20$ given that $\frac{1}{2}$ is a zero of $g(x)$.

(v) Find all zeros of $f(x) = 4x^3 + 19x^2 - 36x - 36$ given that $-\frac{3}{4}$ is a zero of $f(x)$.

12. Write as a single logarithm. Simplify if possible.

a) $3\log_2 a + 2\log_2 b - \frac{1}{4}\log_2 c$

b) $\ln x + 3\ln y$

c) $2\log_3 a + \log_3(a-3) - \log_3(a^2-9)$

d) $3\ln(m) - \ln(m+2) + \frac{3}{2}\ln(m-7)$

e) $\log_5(x-2) + \log_5(x-7) - \log_5(x^2+5x-14)$

Review condensing logarithmic expressions here.

13. Find the exact value of each expression without using a calculator by using properties of logarithms (show your work!).

a) $\log_4 \sqrt[5]{4}$

b) $\ln e^{-10} + \ln e^2$

c) $\log_4 32$

Review properties of logarithms here.

14. Expand, using the properties of logarithms. Simplify if possible.

a) $\log_5 \frac{4xy}{t}$

b) $\log_2 \frac{x^3}{8\sqrt{y}}$

c) $\ln\left(\frac{a^2e^6}{b^{10}}\right)$

Review expanding logarithmic expressions here.

15. Solve each equation. Round the answer to the nearest thousandth, if needed.

a) $3(5^{2x+1}) = 75$ Review

b) $14 + 2^{3a-5} = 197$ Review

c) $\log_2 2x + \log_2(x + \frac{3}{2}) = 1$ Review

d) $e^{2x} - 7e^{x} = 0$ Review (Hint: factor first!!)

e) $\log_9 x + \log_9(2x - 3) = \log_9(x^2 + 4)$ Review

f) $-3(4^{7-2x}) = -48$ Review

g) $\ln(6 - x) = 7$

h) $\log_2(x - 2) + \log_2(x + 5) = 3$ Review

i) $16^{2x-3} = 8^{x+5}$ Review at 5:55 minute marker!

j) $\log_6(x - 3) + \log_6(x - 8) = 2$ Review

16. Evaluate the expression without using a calculator:

a) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

b) $\arcsin 0$

c) $\arccos\left(\cos\left(\frac{\pi}{12}\right)\right)$

d) $\csc\left(\arccos\left(-\frac{3}{4}\right)\right)$

e) $\tan^{-1}(1)$

f) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

g) $\sec\left(\arctan\left(\frac{\sqrt{3}}{3}\right)\right)$

h) $\cot\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

Review inverse trigonometric functions here (for c), here (for a, b, and e), here (for f), and here (for d, g, h at the 5 minute marker)

17. Verify the identities:

a) $\csc\alpha \cdot \tan\alpha = \frac{1}{\cos\alpha}$

b) $\frac{1}{\cos\beta} = \cos\beta + \cos\beta \cdot \tan^2\beta$

c) $\frac{sec^2\,\theta}{1+cot^2\theta} = tan^2\theta$

d) $\frac{cot^2\theta}{\csc\theta+1} = \frac{1-\sin\theta}{\sin\theta}$

Review verifying trigonometric identities here.

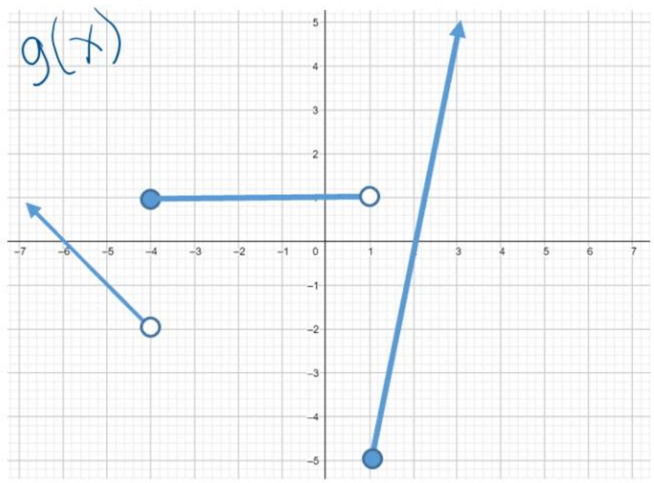
18. (i) Given $5x - 2y = 11$, write the (a) equation of the line that is parallel to it and passes through the point (-10, 3), and (b) equation of the line that is perpendicular to it and passes through the point (-10, 3). Write both equations in slope-intercept form.

(ii) Given $x - 3y = 4$, write the (a) equation of the line that is parallel to it and passes through the point (-6, 1), and (b) equation of the line that is perpendicular to it and passes through the point (-6, 1). Write both equations in slope-intercept form.

(iii) Given $6x + 2y = -13$, write the (a) equation of the line that is parallel to it and passes through the point (-3, -5), and (b) equation of the line that is perpendicular to it and passes through the point (-3, -5). Write both equations in slope-intercept form.

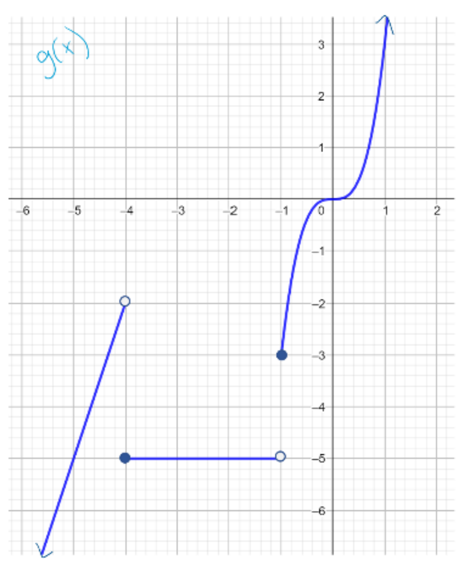
(iv) Given $2x - y = 7$, write the (a) equation of the line that is parallel to it and passes through the point (6, 2), and (b) equation of the line that is perpendicular to it and passes through the point (6, 2). Write both equations in slope-intercept form.

Review writing equations of parallel and perpendicular lines here. Just parallel lines. Just perpendicular lines.



19. (i) Use the graph of $g(x)$ to answer each question.

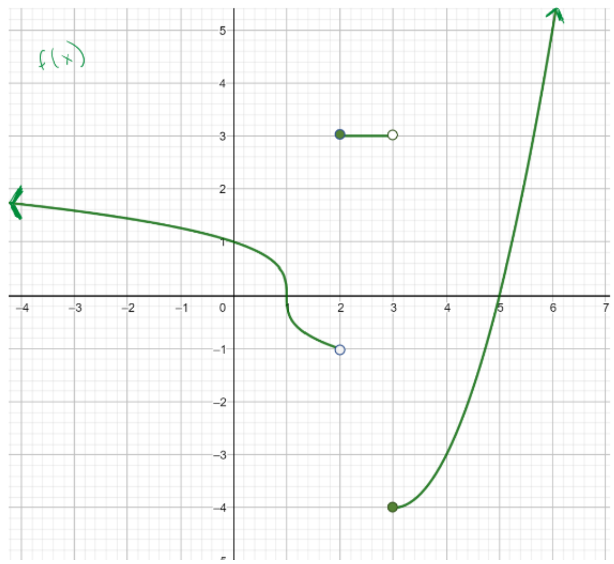
(a) State the domain and range of $g(x)$. Write the answers in interval notation. Review
(b) Find $g(-3)$ and $g(1)$. Review
(c) On which interval(s) is the graph increasing, decreasing, or constant? Review
(d) What are the *x*- and *y*-intercepts of the graph? Review



(ii) Use the graph of $g(x)$ to answer each question.

(a) State the domain and range of $g(x)$. Write the answers in interval notation.
(b) Find $g(-5)$ and $g(-4)$.
(c) On which interval(s) is the graph increasing, decreasing, or constant?
(d) What are the *x*- and *y*-intercepts of the graph?

(iii) Use the graph of $g(x)$ to answer each question.



(a) State the domain and range of $g(x)$. Write the answers in interval notation.

(b) Find $g(3)$ and $g(4)$.

(c) On which interval(s) is the graph increasing, decreasing, or constant?

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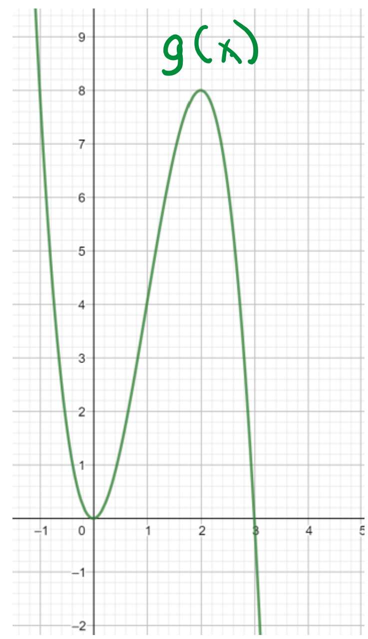
(iv) Use the graph of $g(x)$ to answer each question.



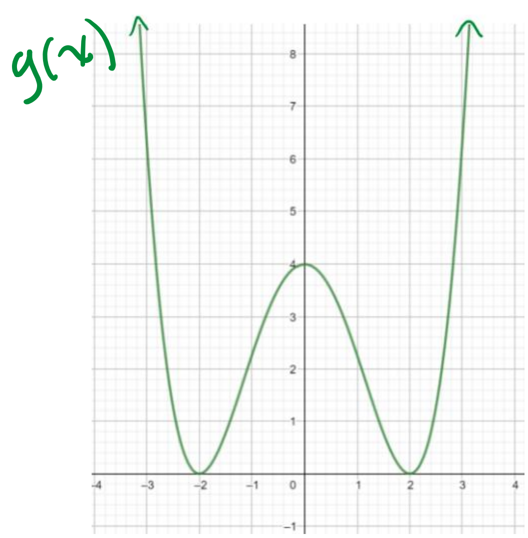
(a) State the domain and range of $g(x)$. Write the answers in interval notation.

(b) Find $g(-1)$ and $g(1)$.

(c) On which interval(s) is the graph increasing, decreasing, or constant?

(d) What are the *x*- and *y*-intercepts of the graph?

(v) Use the graph of $g(x)$ to answer each question.



(a) State the domain and range of $g(x)$. Write the answers in interval notation.

(b) Find $g(-3)$ and $g(2)$.

(c) On which interval(s) is the graph increasing, decreasing, or constant?

(d) What are the *x*- and *y*-intercepts of the graph?

20. (i) Given $f(x) = x^2 - 3x - 4$ and $g(x) = 2x^2 - 8x$, find the following.

a. $(f + g)(x)$
b. $(f - g)(x)$
c. $(fg)(x)$
d. $\frac{f}{g}(x)$, simplify and the domain

(ii) Given $f(x) = \frac{2}{x-4}$ and $h(x) = \frac{4}{x^2-16}$, find the following.

a. $(f + h)(x)$
b. $(f - h)(x)$
c. $(fh)(x)$
d. $\frac{f}{h}(x)$, simplify and the domain

(iii) $f(x) = \frac{5}{x-5}$ and $g(x) = \frac{3}{x^2-25}$, find the following.

a. $(f + g)(x)$
b. $(f - g)(x)$
c. $(fg)(x)$
d. $\frac{f}{g}(x)$, simplify and the domain

Review parts a and b here. Review part d here.

21. Solve the following trigonometric equations. List all possible solutions on the interval $[0, 2\pi)$.

(i) $\sin\theta = \frac{\sqrt{2}}{2}$

(ii) $\tan\theta = \sqrt{3}$

(iii) $2\cos\theta - \sqrt{3} = 0$

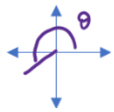
(iv) $2\sin^2\theta + \sin\theta = 0$

(v) $\sec\alpha + \sqrt{2} = 0$

(iv) $\tan^2\alpha - 1 = 0$

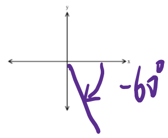
Fall 2022 MAT 206.5 Final Review – Answer Key

1. i)

a.) $210°$ b.) 

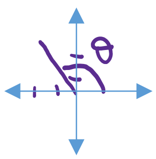
c.) many answers, two are $-\frac{5\pi}{6}$ and $\frac{19\pi}{6}$ (-150° and 570°)

d.) $\hat{\theta} = \frac{\pi}{6}$ (or 30°) e.) $-\frac{\sqrt{3}}{2}$ f.) $\sqrt{3}$

1. ii) a.) $-60°$ b.) 

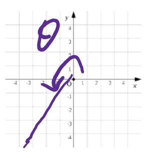
c.) many answers, two are $\frac{5\pi}{3}$ and $-\frac{7\pi}{3}$ (300° and - 420°)

d.) $\hat{\theta} = \frac{\pi}{3}$ (or 60°) e.) $-\frac{\sqrt{3}}{2}$ f.) 2

2. i) 

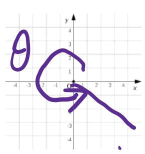
$\tan\theta = -\frac{\sqrt{14}}{2}$ and $\csc\theta = \frac{3\sqrt{7}}{7}$

2. ii)



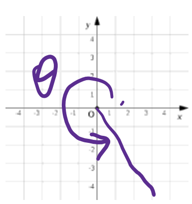
$\sin\theta = -\frac{4\sqrt{21}}{21}$ and $\sec\theta = \frac{-\sqrt{105}}{5}$

2. iii)



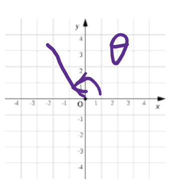
$\sec\theta = \frac{2\sqrt{10}}{5}$ and $\cot\theta = -\frac{\sqrt{15}}{3}$

2. iv)



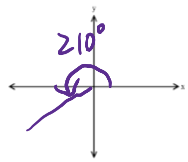
$\csc\theta = -\frac{7\sqrt{10}}{20}$ and $\tan\theta = -\frac{2\sqrt{10}}{3}$

2. v)

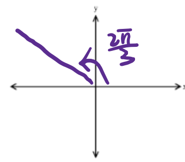


$\csc\theta = \frac{\sqrt{35}}{5}$ and $\sec\theta = -\frac{\sqrt{14}}{2}$

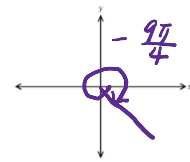
3a.) $\frac{\sqrt{3}}{3}$



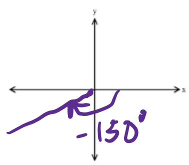
3b.) $-\frac{1}{2}$



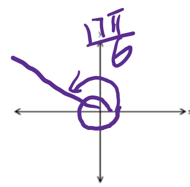
3c.) $-\frac{\sqrt{2}}{2}$



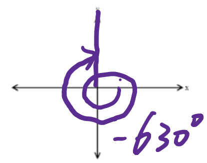
3d.) $\sqrt{3}$



3e.) $-\frac{2\sqrt{3}}{3}$



3f.) 1



4. i) a.) $(-\infty, \infty)$ b.) $(-\infty, 3]$ c.) $-x-1$ d.) $(-\infty, 3]$

4. ii) a.) $(-\infty, 4) \cup (4, \infty)$ b.) $(-\infty, \infty)$ c.) $\frac{1}{(x-4)^2} - 16$ d.) $(-\infty, 4) \cup (4, \infty)$

4. iii) a.) $(-\infty, \infty)$ b.) $[-2, \infty)$ c.) $x-1$ d.) $[-2, \infty)$

4. iv) a.) $(-\infty, 7) \cup (7, \infty)$ b.) $(-\infty, 3]$ c.) $\frac{1}{\sqrt{3-x}-7}$ d.) $(-\infty, -46) \cup (-46, 3]$

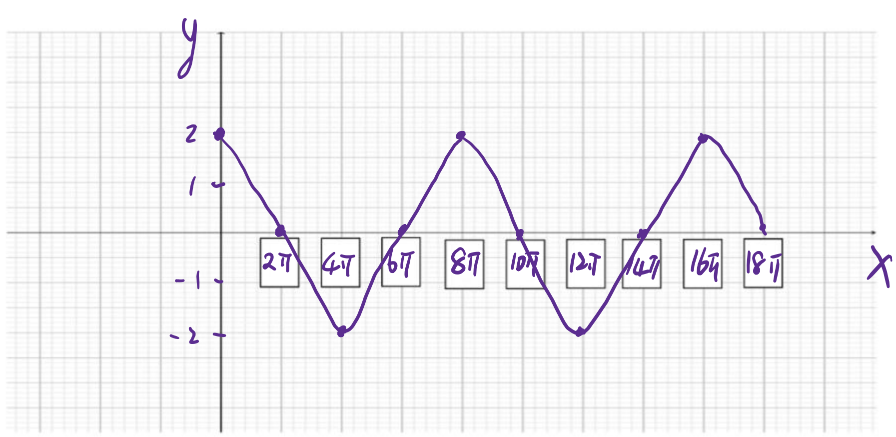
4. v) a.) $(-\infty, \infty)$ b.) $(-3, \infty)$ c.) $\log_2(x-2)$ d.) $(2, \infty)$

5. i) a.) 8π b.) A = 2 c.) Phase shift = 0 d.) $y = 0$

e.)

x	0	2π	4π	6π	8π
$f(x)$	2	0	-2	0	2

f)

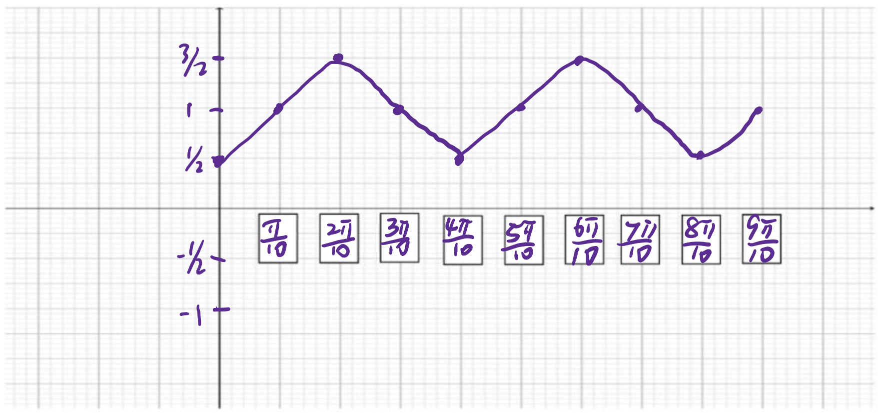


5. ii) a.) $\frac{2\pi}{5}$ b.) A = $\frac{1}{2}$ c.) Phase shift = 0 d.) $y = 1$

e.)

x	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$ (or $\frac{2\pi}{10}$)	$\frac{3\pi}{10}$	$\frac{2\pi}{5}$ (or $\frac{4\pi}{10}$)
$f(x)$	$\frac{1}{2}$	1	$\frac{3}{2}$	1	$\frac{1}{2}$

f)

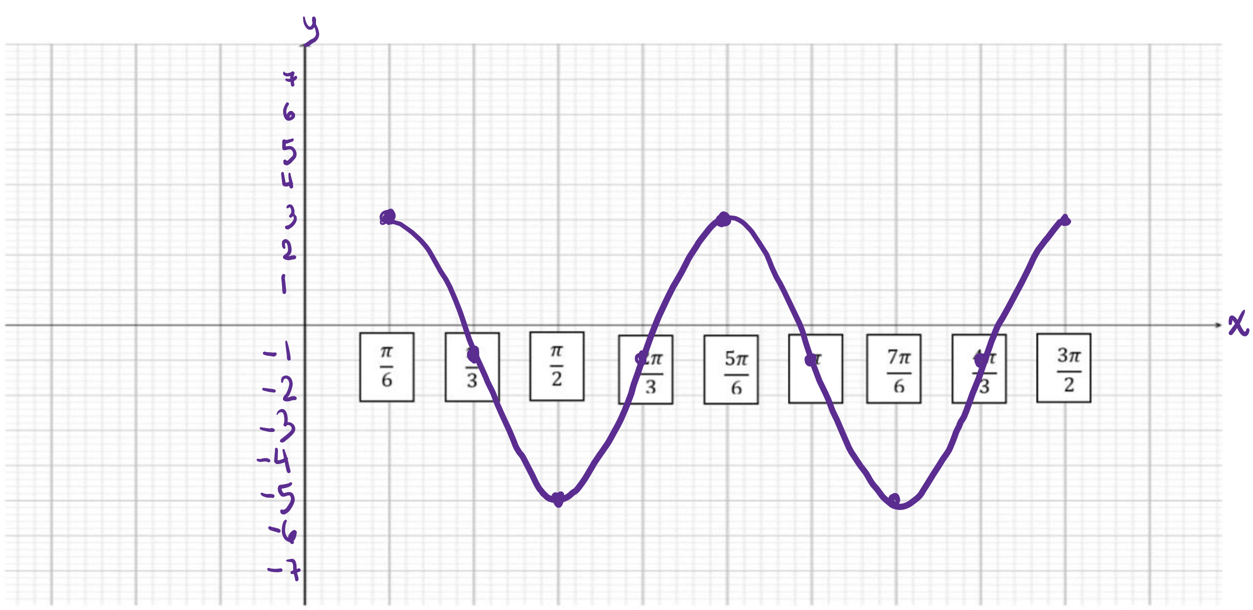


5. iii) a.) $\frac{2\pi}{3}$ b.) A = 4 c.) Phase shift = $\frac{\pi}{6}$ d.) $y = -1$

e.)

x	$\frac{\pi}{6}$	$\frac{\pi}{3}$ (or $\frac{2\pi}{6}$)	$\frac{\pi}{2}$ (or $\frac{3\pi}{6}$)	$\frac{2\pi}{3}$ (or $\frac{4\pi}{6}$)	$\frac{5\pi}{3}$
$f(x)$	3	-1	-5	-1	3

f)

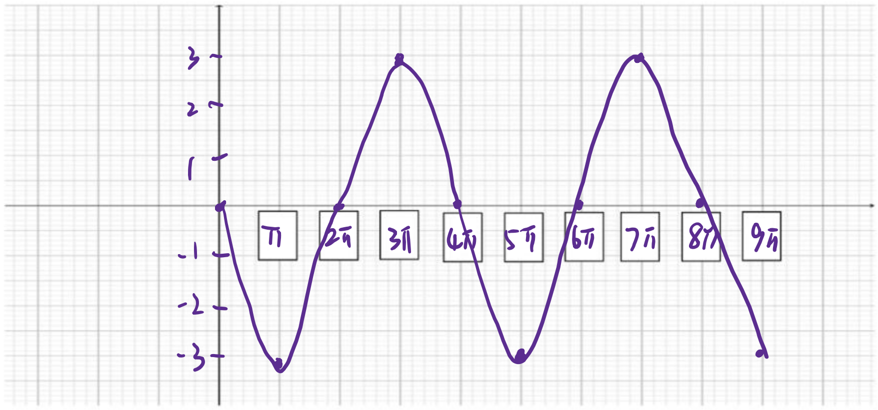


5. iv) a.) 4π b.) A = 3 c.) Phase shift = 0 d.) $y = 0$

e.)

f)

x	0	π	2π	3π	4π
$f(x)$	0	-3	0	3	0

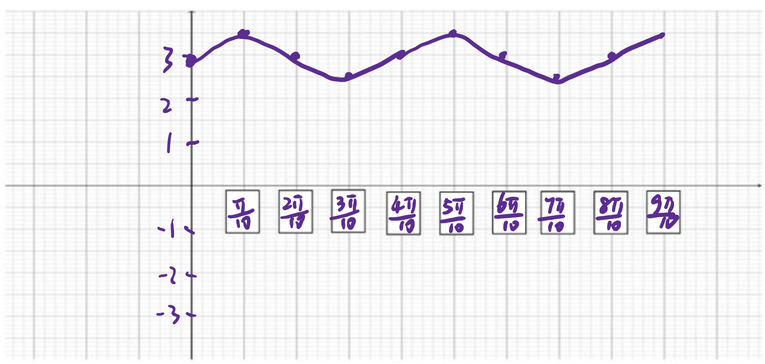


5. v) a.) $\frac{2\pi}{5}$ b.) A = $\frac{1}{2}$ c.) Phase shift = 0 d.) $y = 3$

e.)

f)

x	0	$\frac{\pi}{10}$	$\frac{\pi}{5}$ (or $\frac{2\pi}{10}$)	$\frac{3\pi}{10}$	$\frac{2\pi}{5}$ (or $\frac{4\pi}{10}$)
$f(x)$	3	3.5	3	2.5	3

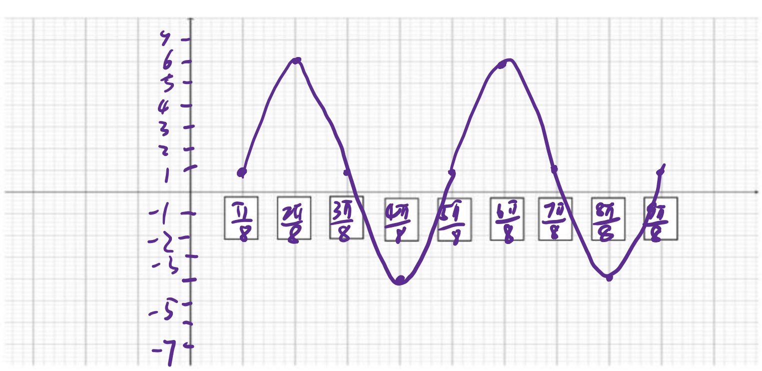


5. vi) a.) $\frac{\pi}{2}$ b.) A = 5 c.) Phase shift = $\frac{\pi}{8}$ d.) $y = 1$

e.)

f.)

x	$\frac{\pi}{8}$	$\frac{\pi}{4}$ (or $\frac{2\pi}{8}$)	$\frac{3\pi}{8}$	$\frac{\pi}{2}$ (or $\frac{4\pi}{8}$)	$\frac{5\pi}{8}$
$f(x)$	1	6	1	-4	1



6. i) a.) $\frac{\pi}{2}$ b.) 2 c.) $\frac{\pi}{8}$ d.) $y = -3$

e.)

x	$\frac{\pi}{8}$	$\frac{\pi}{4}$ or $\frac{2\pi}{8}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$ or $\frac{4\pi}{8}$	$\frac{5\pi}{8}$
$g(x)$	-3	-1	-3	-5	-3

f.) $g(x) = 2\sin\left(4x - \frac{\pi}{2}\right) - 3$

6. ii) a.) π b.) $\frac{1}{2}$ c.) $\frac{5\pi}{8}$ d.) $y = 3$

e.)

x	$\frac{5\pi}{8}$	$\frac{7\pi}{8}$	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$
$g(x)$	3.5	3	2.5	3	3.5

f.) $g(x) = \frac{1}{2}\cos\left(2x - \frac{5\pi}{4}\right) + 3$

7.) (i) $2h - 15$ (ii) 2 (iii) $-\frac{1}{(x+h+6)(x+6)}$ (iv) 0 (v) -2x+4-h (vi) $-\frac{5}{4(h+4)}$

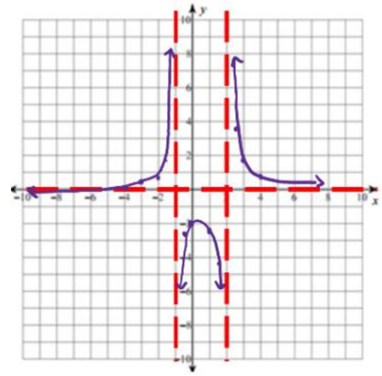
8i.) Domain: $(-\infty, -8) \cup (-8, 5) \cup (5, \infty)$

Hole located at $\left(5, \frac{10}{13}\right)$

Vertical asymptote at x = -8

Horizontal asymptote at y = 1

8ii.) Domain: $(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

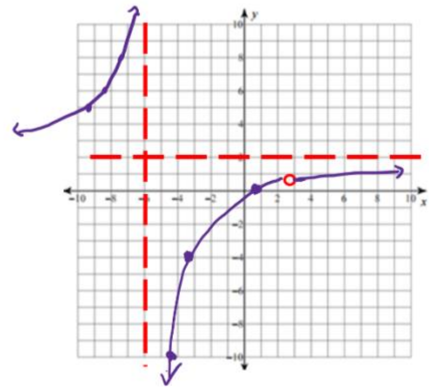


No hole

Vertical asymptotes at x = -1 and x = 2

Horizontal asymptote at y = 0

8iii) Domain: $(-\infty,-6)\cup(-6,3)\cup(3,\infty)$

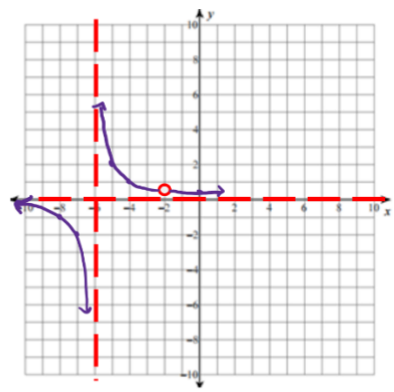


Hole at $\left(3,\frac{2}{3}\right)$

Vertical asymptote at x = -6

Horizontal asymptote at y = 2

8iv) Domain: $(-\infty,-6)\cup(-6,-2)\cup(-2,\infty)$

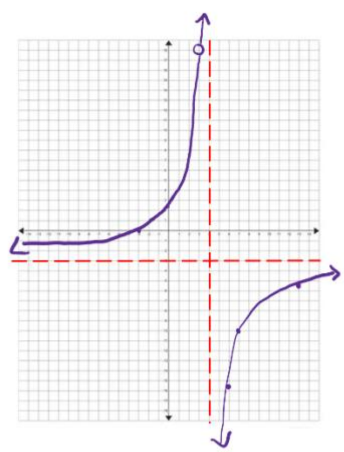


Hole at $\left(-2,\frac{1}{2}\right)$

Vertical asymptote at x = -6

Horizontal asymptote at y = 0

8v) Domain: $(-\infty,3)\cup(3,4)\cup(4,\infty)$



Hole at $(3,18)$

Vertical asymptote at x = 4

Horizontal asymptote at y = -3

9a.) $\frac{1}{x}-\frac{1}{x+1}$ 9b.) $\frac{1}{x-3}-\frac{1}{x}$ 9c.) $\frac{1}{x-2}-\frac{1}{x+3}$

9d.) $\frac{1}{5(x+2)}+\frac{4}{5(x-3)}$ 9e.) $\frac{2}{x-1}-\frac{1}{(x-1)^2}$

10i.) x = {-2, -3, 5} 10ii.) x = {-5, 1, 3} 10iii.) x = {-4, -1, 5} 10iv.) x = $\{-\frac{1}{2}, 1, 6\}$

11i.) $x=\left\{\frac{2}{3}, 4i, -4i\right\}$ ii.) $x=\left\{-3,-2,-\frac{1}{3},3\right\}$ iii.) $x=\left\{\frac{1}{5}, 3i, -3i\right\}$

iv.) $x=\left\{-2,\frac{1}{2},2,5\right\}$ v.) $x=\left\{-2,\frac{3}{4},-2i,2i\right\}$

12a.) $\log_2\left(\frac{a^3b^2}{\sqrt[4]{c}}\right)$ 12b.) $\ln(xy^3)$ 12c.) $\log_3\left(\frac{a^2}{a+3}\right)$

12d.) $\ln\left(\frac{m^3\cdot\sqrt[3]{(m-7)^2}}{m+2}\right)$ 12e.) $\log_5\left(\frac{x-7}{x+7}\right)$

13a.) $\frac{1}{5}$ 13b.) -8 13c.) $\frac{5}{2}$

14a.) $\log_5(4)+\log_5(x)+\log_5(y)-\log_5(t)$ 14b.) $3\log_2 x-3-\frac{1}{2}\log_2 y$

14c.) $2\ln(a)+6-10\ln(b)$

15a.) $x=\frac{1}{2}$ 15b.) $x\approx 4.172$ 15c.) $x=\frac{1}{2}$

15d.) $x\approx 1.946$ 15e.) x = 4 15f.) $x=\frac{5}{2}$ 15g.) $x\approx -1090.63$ 15h.) $x=3$

15i.) $x=\frac{27}{5}=5\frac{2}{5}$ 15j.) $x=12$

16a.) $\frac{3\pi}{4}$ 16b.) 0 16c.) $\frac{\pi}{12}$ 16d.) $\frac{4\sqrt{7}}{7}$ 16e.) $\frac{\pi}{4}$ 16f.) $\frac{\pi}{3}$

16g.) $\frac{2\sqrt{3}}{3}$ 16h.) $\frac{\sqrt{5}}{2}$

17a.) There are many correct answers! This is one correct answer.

$$\frac{1}{\sin\alpha} \cdot \frac{\sin\alpha}{\cos\alpha}$$

$$= \frac{1}{\cos\alpha}$$

17b.) There are many correct answers! This is one correct answer.

$$\cos\beta \cdot (1 + \tan^2\beta)$$

$$= \cos\beta \cdot \sec^2\beta$$

$$= \cos\beta \cdot \frac{1}{\cos^2\beta}$$

$$= \frac{1}{\cos\beta}$$

17c.) There are many correct answers! This is one correct answer.

$$\Rightarrow \frac{\sec^2\theta}{\csc^2\theta} = \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

17d.) There are many correct answers! This is one correct answer.

$$\frac{\csc^2\theta - 1}{\csc\theta + 1}$$

$$\Rightarrow \frac{(\csc\theta + 1)(\csc\theta - 1)}{\csc\theta + 1}$$

$$\Rightarrow \csc\theta - 1$$

$$= \frac{1}{\sin\theta} - 1 \Rightarrow \frac{1}{\sin\theta} - \frac{\sin\theta}{\sin\theta}$$

$$= \frac{1 - \sin\theta}{\sin\theta}$$

18i.) Equation of parallel line: $y = \frac{5}{2}x + 28$

Equation of perpendicular line: $y = -\frac{2}{5}x - 1$

18ii.) Equation of parallel line: y =$\frac{1}{3}$ x + 3

Equation of the perpendicular line: y= -3x -17

18iii.) Equation of the parallel line: y= 2x + 1

Equation of the perpendicular line: Y =$\frac{-1}{2}$x - $\frac{13}{2}$

18iv.) Equation of the paralle line: y = 2x -10

Equation of the perpendicular line: y = $\frac{-1}{2}$x + 5

19ia.) Domain: $(-\infty, \infty)$; Range: $[-5, \infty)$ 19ib.) $g(-3) = 1$; $g(1) = -5$

19ic.) Increasing: $[1, \infty)$; Decreasing: $(-\infty, -4)$; Constant: $[-4, 1)$

19id.) x-intercepts: (-6, 0) and (2, 0); y-intercept: (0, 1)

19iia.) Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$ 19iib.) $g(-5) = -5$; $g(-4) = -5$

19iic.) Increasing: $(-\infty, -4) \cup [-1, \infty)$; decreasing: none; constant: [-4, -1]

19iid.) x-intercept = y-intercept = (0, 0)

19iiia.) Domain: $(-\infty, \infty)$; Range: $[-4, \infty)$ 19iiib.) $g(3) = -4$; $g(4) = -3$

19iiic.) Increasing: $[3, \infty)$; Decreasing: $(-\infty, 2)$; Constant: $[2, 3)$

19iiid.) x-intercepts: (1, 0) and (5, 0); y-intercept: (0, 1)

19iva.) Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$ 19ivb.) $g(-1) = 8$; $g(1) = 4$

19ivc.) Increasing: $(0, 2)$; Decreasing: $(-\infty, 0) \cup (2, \infty)$; Constant: none

19ivd.) x-intercepts: (-6, 0) and (2, 0); y-intercept: (0, 1)

19va.) Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

19vb.) $g(-3) = 6$; $g(2) = 0$

19vc.) Increasing: $(-2, 0) \cup (2, \infty)$; Decreasing: $(-\infty, -2) \cup (0, 2)$; Constant: none

19id.) x-intercepts: (-2, 0) and (2, 0); y-intercept: (0, 4)

20ia.) $3x^2 - 11x - 4$

20ib.) $-x^2 + 5x - 4$

20ic.) $20x^4 - 14x^3 + 16x^2 + 32x$

20id.) $\frac{x+1}{2x}$ and the domain is $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$

20iia) $\frac{2x+12}{x^2-16}$

20iib.) $\frac{2x+4}{x^2-16}$

20iic.) $\frac{8}{(x-4)^2(x+4)}$

20iid.) $\frac{x+4}{2}$ and the domain is $(-\infty, -4) \cup (-4,4) \cup (4, \infty)$

20iiia.) $\frac{5x+28}{x^2-25}$

20iiib.) $\frac{5x+22}{x^2-25}$

20iiic.) $\frac{15}{(x-5)^2(x+5)}$

20iiid.) $\frac{5x+25}{3}$ and the domain is $(-\infty, -5) \cup (-5, 5) \cup (5, +\infty)$

21i.) $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

21ii.) $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

21iii.) $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

21iv.) $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

21v.) $\alpha = \frac{5\pi}{4}$

21vi.) $\alpha = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$