

# BOROUGH OF MANHATTAN COMMUNITY COLLEGE

## Math Lab

### Practice Problems for the MAT 206 Exemption Exam

The examination to be exempt from MAT 206 will consist of 10 questions. Please do not assume that the content or difficulty level of these practice questions are exactly the same as the actual examination. We suggest you study the range of topics described on the syllabus as well as the types of problem on this practice.

1. Graph the following functions by completing the square method. Find the vertex, axes of symmetry, and x-intercepts or y-intercepts (if possible).

a.  $f(x) = -x^2 - 2x + 3$

b.  $f(x) = -2x^2 + 10x$

c.  $f(x) = 2x^2 - 7x - 30$

d.  $f(x) = \frac{1}{4}x^2 - 2x - 12$

e.  $f(x) = 3x^2 - 4x + 3$

2. Graph the following functions. Find the domain and range of each function.

a.  $f(x) = 10 - \sqrt{x+5}$

b.  $f(x) = \sqrt{x-2} - 3$

c.  $f(x) = -4x^2 + 8x + 9$

d.  $f(x) = 2^{x-2} - 4$

e.  $f(x) = |x - 2| + 3$

f.  $f(x) = \log_2(x + 4) - 1$

g.  $f(x) = 2\sin\left(\frac{x}{2} - \frac{3\pi}{8}\right) + 1$

h.  $f(x) = \frac{3x+2}{x-1}$

3. Find the inverse function. Verify that  $f(x)$  and your result are inverse functions.

a.  $f(x) = 5x - 3$

b.  $f(x) = x^3 + 10$

c.  $f(x) = \frac{x-1}{x+4}$

d.  $f(x) = \sqrt{2x - 7}$

e.  $f(x) = \sqrt[3]{3x + 8}$

4. Find the zeroes of the polynomial.

a.  $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$

b.  $f(x) = x^3 - 7x + 6$

c.  $f(x) = x^4 - 4x^3 + 7x^2 - 16x + 12$

d.  $f(x) = 3x^3 + 2x^2 - 19x + 6$

5. Solve for x.

a.  $5^{x+3} = 25$

b.  $x + 1 = 3 - \sqrt{x + 10}$

c.  $\log(x + 5)^2 = 3$

d.  $\log_6 x + \log_6(x - 1) = 1$

e.  $\sin^2 x = \frac{1}{2} \tan^2 x$

f.  $2 = 3\cos^2 x$

6. Rewrite each angle in radian measure.

a.  $30^\circ$

b.  $315^\circ$

c.  $-144^\circ$

d.  $-270^\circ$

e.  $-225^\circ$

7. Rewrite each angle in degree measure.

a.  $\frac{3\pi}{2}$

b.  $-\frac{7\pi}{3}$

c.  $-\frac{\pi}{9}$

d.  $\frac{5\pi}{4}$

e.  $\frac{34\pi}{15}$

8. Find exact value of each expression.

a.  $\cos(165^\circ)$

b.  $\sin(15^\circ)$

c.  $\sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

d.  $\cos\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$

9. Find the exact value of the six trigonometric functions using the given information.

a.  $\sin\theta = \frac{5}{13}, \frac{\pi}{2} \leq \theta \leq \pi$

b.  $\tan\theta = 5, \pi \leq \theta \leq \frac{3\pi}{2}$

c.  $\cos\theta = \frac{14}{17}, \frac{3\pi}{2} \leq \theta \leq 2\pi$

10. Verify the identity of the following:

a.  $\frac{\sin^2 x - 1}{\sec^2 x} = -\cos^4 x$

b.  $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$

c.  $\frac{\cot^2 \theta}{1 + \csc \theta} = \frac{1 - \sin \theta}{\sin \theta}$

d.  $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

11. Find vertical or horizontal asymptotes (if possible) of the following functions.

a.  $f(x) = \frac{1}{2x - 5}$

b.  $f(x) = \frac{2x^2 + 4x}{x^2 - 3x - 10}$

c.  $f(x) = e^{x-3} + 2$

d.  $f(x) = \ln(x - e) + 2$

12. Write the partial fraction decomposition of each expression.

a.  $\frac{x+7}{x^2-x-6}$

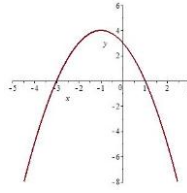
b.  $\frac{x^4+2x^3+6x^2+20x+6}{x^3+2x^2+x}$

c.  $\frac{3x^2+4x+4}{x^3+4x}$

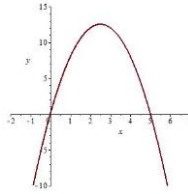
d.  $\frac{8x^3+13x}{(x^2+2)^2}$

1.

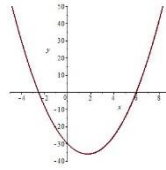
- a.  $f(x) = -(x + 1)^2 + 4$   
 Vertex:  $(-1, 4)$   
 Axis of symmetry:  $x = -1$   
 x-intercepts:  $(1, 0)$  and  $(-3, 0)$   
 y-intercept:  $(0, 3)$



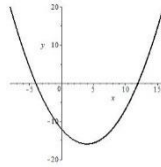
- b.  $f(x) = -2\left(x - \frac{5}{2}\right)^2 + \frac{25}{2}$   
 Vertex:  $\left(\frac{5}{2}, \frac{25}{2}\right)$   
 Axis of symmetry:  $x = \frac{5}{2}$   
 x-intercepts:  $(0, 0)$  and  $(5, 0)$   
 y-intercept:  $(0, 0)$



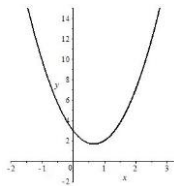
- c.  $f(x) = 2\left(x - \frac{7}{4}\right)^2 - \frac{289}{8}$   
 Vertex:  $\left(\frac{7}{4}, \frac{289}{8}\right)$   
 Axis of symmetry:  $x = \frac{7}{4}$   
 x-intercepts:  $\left(-\frac{5}{2}, 0\right)$  and  $(6, 0)$   
 y-intercept:  $(0, -30)$



- d.  $f(x) = \frac{1}{4}(x - 4)^2 - 16$   
 Vertex:  $(4, -16)$   
 Axis of symmetry:  $x = 4$   
 x-intercepts:  $(-4, 0)$  and  $(12, 0)$   
 y-intercept:  $(0, -12)$

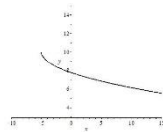


- e.  $f(x) = 3\left(x - \frac{2}{3}\right)^2 + \frac{5}{3}$   
 Vertex:  $\left(\frac{2}{3}, \frac{5}{3}\right)$   
 Axis of symmetry:  $x = \frac{2}{3}$   
 x-intercepts: None  
 y-intercept:  $(0, 3)$

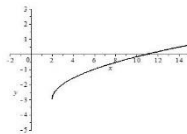


2.

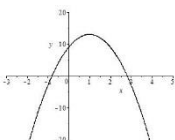
- a. Domain:  $[-5, \infty)$   
 Range:  $(-\infty, 10]$



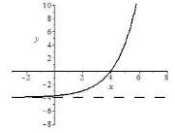
- b. Domain:  $[-5, \infty)$   
 Range:  $(-\infty, 10]$



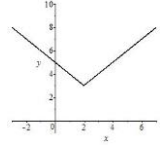
- c. Domain:  $(-\infty, \infty)$   
 Range:  $(-\infty, 13]$



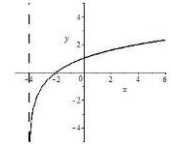
- d. Domain:  $(-\infty, \infty)$   
 Range:  $(-4, \infty)$



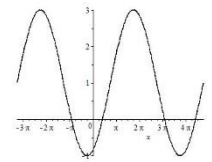
- e. Domain:  $(-\infty, \infty)$   
 Range:  $[3, \infty)$



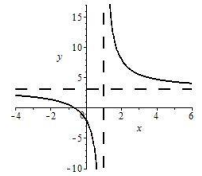
- f. Domain:  $(-4, \infty)$   
 Range:  $(-\infty, \infty)$



- g. Domain:  $(-\infty, \infty)$   
 Range:  $[-1, 3]$



- h. Domain:  $(-\infty, 1) \cup (1, \infty)$   
 Range:  $(-\infty, \frac{3}{2}) \cup (\frac{3}{2}, \infty)$



3.

- a.  $f^{-1}(x) = \frac{x+3}{5}$   
 b.  $f^{-1}(x) = \sqrt[3]{x-10}$   
 c.  $f^{-1}(x) = \frac{4x+1}{1-x}$   
 d.  $f^{-1}(x) = \frac{x^2+7}{2}$   
 e.  $f^{-1}(x) = \frac{x^3-8}{3}$

4.

- a. Zeros:  $x = 2, -1, -3, -\frac{3}{2}$   
 b. Zeros:  $x = -3, 1, 2$   
 c. Zeros:  $x = 1, 3, \pm 2i$   
 d. Zeros:  $x = 2, -3, \frac{1}{3}$

5.

- a.  $x = -1$   
 b.  $x = -1$  ( $x = 6$  is **not**)  
 c.  $x = -5 - 10\sqrt{10}$ ,  $x = -5 + 10\sqrt{10}$

d.  $x = 3$  ( $x = -2$  is **not**)

e.  $x = k\pi, -\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi, k \in \mathbb{Z}$

f.  $x = -\arccos\left(\frac{\sqrt{6}}{3}\right) + 2k\pi, k \in \mathbb{Z}$

$x = \pi + \arccos\left(\frac{\sqrt{6}}{3}\right) + 2k\pi, k \in \mathbb{Z}$

c.  $\sin\theta = -\frac{\sqrt{93}}{17}, \quad \csc\theta = -\frac{17\sqrt{93}}{17}$   
 $\cos\theta = \frac{14}{17}, \quad \sec\theta = \frac{17}{14}$   
 $\tan\theta = -\frac{\sqrt{93}}{14}, \quad \cot\theta = -\frac{14\sqrt{93}}{93}$

10. Try.

6.

a.  $\frac{\pi}{6}$     b.  $\frac{7\pi}{4}$     c.  $-\frac{4\pi}{5}$     d.  $-\frac{3\pi}{2}$   
 e.  $-\frac{5\pi}{4}$

7.

a.  $270^\circ$     b.  $-420^\circ$     c.  $-20^\circ$   
 d.  $225^\circ$     e.  $408^\circ$

8.

a.  $-\frac{\sqrt{2}+\sqrt{6}}{4}$     b.  $\frac{\sqrt{6}-\sqrt{2}}{4}$   
 c.  $\frac{\sqrt{2}+\sqrt{6}}{4}$     d.  $-\sqrt{3}$

9.

a.  $\sin\theta = \frac{5}{13}, \quad \csc\theta = \frac{13}{5}$   
 $\cos\theta = -\frac{12}{13}, \quad \sec\theta = -\frac{13}{12}$   
 $\tan\theta = -\frac{5}{12}, \quad \cot\theta = -\frac{12}{5}$

b.  $\sin\theta = \frac{5\sqrt{26}}{26}, \quad \csc\theta = -\frac{\sqrt{26}}{5}$   
 $\cos\theta = -\frac{\sqrt{26}}{26}, \quad \sec\theta = -\sqrt{26}$   
 $\tan\theta = 5, \quad \cot\theta = \frac{1}{5}$

11.

a. V.A.:  $x = \frac{5}{2}$     H.A.:  $y = 2$   
 b. V.A.:  $x = 5$     H.A.:  $y = 2$   
 c. V.A.: None    H.A.:  $y = 2$   
 d. V.A.:  $x = e$     H.A.: None

12.

a.  $-\frac{1}{x+2} + \frac{2}{x-3}$   
 b.  $x + \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}$   
 c.  $\frac{1}{x} + \frac{2x+4}{x^2+4}$   
 d.  $\frac{8x}{x^2+2} - \frac{3x}{(x^2+2)^2}$